Numerical Simulation of Shielding Current Density in High-Temperature Superconducting Film: Influence of Film Edge on Permanent Magnet Method

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Abstract —A numerical code for analyzing the shielding current density has been developed on the basis of the modified constitutive-relation method. By means of the code, the permanent-magnet method for measuring the critical current density has been investigated. The results of computations show that, if a magnet is placed near the edge of a high-temperature superconducting film, the accuracy of the permanent-magnet method is degraded remarkably.

I. Introduction

The critical current density is one of the most important parameters for engineering applications of a high-temperature superconducting (HTS) film. Ohshima *et al.* [1] developed a contactless and nondestructive method for measuring the critical current density j_C of an HTS film. While bringing a permanent magnet closer to an HTS film, they measured the electromagnetic force acting on the film. As a result, they found that the maximum repulsive force is approximately proportional to j_C. This tendency implies that j_C can be determined by measuring the electromagnetic interaction between the magnet and the film. This method is called the permanent-magnet method.

In order to simulate the permanent-magnet method, the time evolution of the shielding current density has to be determined numerically. Although the implicit method has been so far applied to the initial-boundary-value problem of the shielding current density, it costs enormous CPU time [2], [3]. In addition, the problem cannot be always solved with the Runge-Kutta method even when an adaptive step-size control (ASSC) algorithm is incorporated. This is mainly because the shielding current density diverges in the ASSC algorithm. In order to resolve this difficulty, the authors proposed the modified constitutive-relation method [4]. In the method, the *J-E* constitutive relation is modified so as not to change the solution. The results of numerical experiments show that, by using the modified constitutive-relation method, the divergence of the shielding current density can be completely suppressed [4].

The purpose of the present study is to numerically investigate the permanent-magnet method by means of a non-axisymmetric code in which the modified constitutive-relation method is implemented. Especially, the present study focuses on the influence of a film edge on the accuracy of the permanent-magnet method.

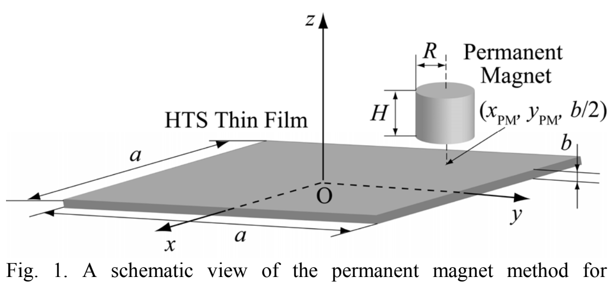


Fig. 1. A schematic view of the permanent magnet method for measuring the critical current density in an HTS film.

II. Governing Equations

In the permanent-magnet method, a cylindrical permanent magnet is placed above an HTS film so that the symmetry axis of the magnet may be vertical to the film surface (see Fig. 1). While the magnet is first brought closer to the film and it is subsequently moved away from the film, the electromagnetic force acting on the film is measured. In order to simulate the movement of the magnet, the distance L between the magnet and the film is assumed as follows: it is first reduced from $L = L_{max}$ to $L = L_{min}$ at a speed $v = (L_{max} - L_{min})/\tau_0$ and, just after that, it is increased from $L = L_{min}$ to $L = L_{max}$ at the same speed.

For simplicity, an HTS film is assumed to have a square cross section Ω through the thickness. Throughout the present study, the side length of Ω and the thickness of the film are denoted by a and b, respectively, and the boundary of Ω is represented by $\partial\Omega$. In addition, the radius and the height of the magnet are denoted by R and H, respectively. By choosing the centroid of the film as the origin and taking the thickness direction of the film as z-axis, let us use the Cartesian coordinate system. In terms of the coordinate system, the symmetry axis of the magnet can be written as $(x, y) = (x_{PM}, y_{PM})$. In the following, $\boldsymbol{x}$ and $\boldsymbol{x}'$ are position vectors of two points in the xy plane.

As the *J-E* constitutive relation, we adopt the power law [4]-[6]:

$$\boldsymbol{E} = E(|\,\boldsymbol{j}\,|)[\boldsymbol{j}/|\,\boldsymbol{j}\,|], \tag{1}$$

$$E(j) = E_C[j/j_C]^N, \tag{2}$$

where $\boldsymbol{j}$ and $\boldsymbol{E}$ denote the shielding current density and the electric field, respectively. In addition, j_C and E_C are the critical current density and the critical electric field, respectively, and N is a positive constant.

Under the thin-layer approximation, there exists a scalar function $S(\boldsymbol{x}, t)$ such that $\boldsymbol{j} = (2/b)\nabla\times(S\boldsymbol{e}_z)$ and its temporal variation is governed by the following integro-differential equation [2]-[4]:

$$\mu_0 \frac{\partial}{\partial t}(\hat{W}S) = -\frac{\partial}{\partial t}\langle \boldsymbol{B}\cdot\boldsymbol{e}_z \rangle - (\nabla\times\boldsymbol{E})\cdot\boldsymbol{e}_z. \tag{3}$$

Here, $\boldsymbol{B}$ denotes the applied magnetic flux density and $\langle\ \rangle$ denotes an average operator through the thickness. In addition, $\hat{W}$ is an operator defined by

$$\hat{W}S \equiv \iint_{\Omega} Q(|\boldsymbol{x}-\boldsymbol{x}'|)S(\boldsymbol{x}',t)\, d^2\boldsymbol{x}' + \frac{2}{b}S(\boldsymbol{x},t), \qquad (4)$$

where $Q(r) \equiv -(\pi b^2)^{-1}[r^{-1}-(r^2+b^2)^{-1/2}]$.

The initial and boundary conditions to (3) are assumed as follows: $S = 0$ at $t = 0$ and $S = 0$ on $\partial\Omega$. By solving the initial-boundary-value problem of (3), we can investigate the time evolution of the shielding current density. Throughout the present study, the physical and geometrical parameters are fixed as follows: $a = 40$ mm, $b = 250$ nm, $E_C = 0.1$ mV/m, $L_{max} = 20$ mm, $L_{min} = 0.5$ mm, $\tau_0 = 39$ s, $R = 2.5$ mm, $H = 3$ mm, and $x_{PM} = 0$ mm.

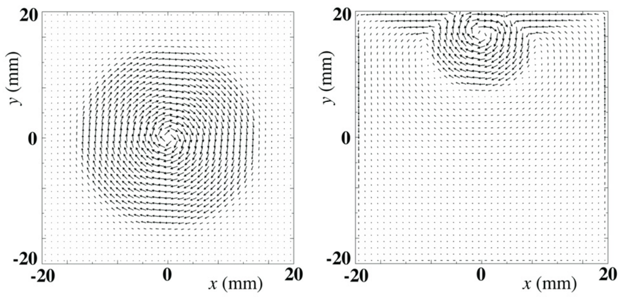
III. Simulation of Permanent-Magnet Method

On the basis of the modified constitutive-relation method [4], a numerical code has been developed for analyzing the time evolution of the shielding current density. In this section, we investigate the permanent-magnet method by use of the code.

In order to numerically reproduce the permanent-magnet method, the electromagnetic force F_z acting on the HTS film is evaluated as a function of L. By extrapolating the resulting F_z-L curve, we can easily determine the approximate value of F_z for $L = 0$. The value is called a maximum repulsive force and is denoted by F_M.

Let us first investigate how the magnet position affects the spatial distribution of the shielding current density. To this end, the $\boldsymbol{j}$-distributions are determined for various values of y_{PM}. Typical examples are shown in Figs. 2(a) and 2(b). For the case with $y_{PM} = 0$ mm, the $\boldsymbol{j}$-distribution is almost axisymmetric around the symmetry axis of the magnet and, hence, the edge effect on $\boldsymbol{j}$ is not at all observed (see Fig. 2(a)). In contrast, for the case with $y_{PM} = 17$ mm, the $\boldsymbol{j}$-distribution is strongly influenced by the film edge so that the axisymmetry is broken considerably (see Fig. 2(b)). Hence, the proportional relationship between j_C and F_M could not be expected when the magnet is placed near the film edge.

Next, we investigate the influence of the magnet position on the proportional relationship between j_C and F_M. The dependence of F_M on j_C is numerically determined for various values of y_{PM} and is depicted in Fig. 3. This figure indicates that, regardless of the magnet position, j_C is in proportion to F_M. Furthermore, the proportionality constant C ($=b\, j_C/F_M$) depends on the coil position (x_{PM}, y_{PM}). For the purpose of investigating the dependence quantitatively, C is evaluated as a function of y_{PM} and is plotted in Fig. 4. The proportional constant C remains almost constant for $y_{PM} < 15$ mm, whereas it rapidly develops with y_{PM} for $y_{PM} > 15$ mm. This result means that the accuracy of the permanent-magnet method is remarkably degraded near the film edge.



(a) (b)

Fig. 2. Spatial distributions of the shielding current density at $t = \tau_0$ for the case with $j_C = 3.2$ MA/cm^2. Here, (a) $y_{PM} = 0$ mm and (b) $y_{PM} = 17$ mm.

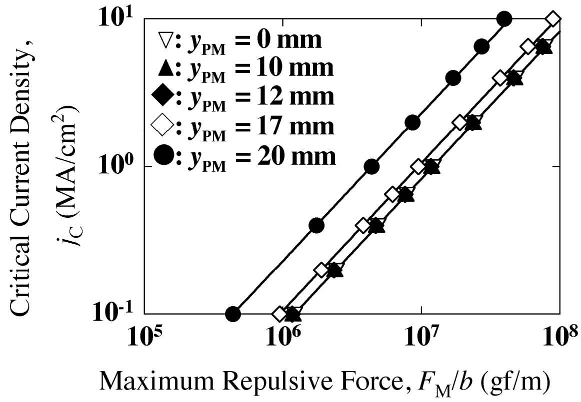


Fig. 3. Dependence of the maximum repulsive force F_M on the critical current density j_C.

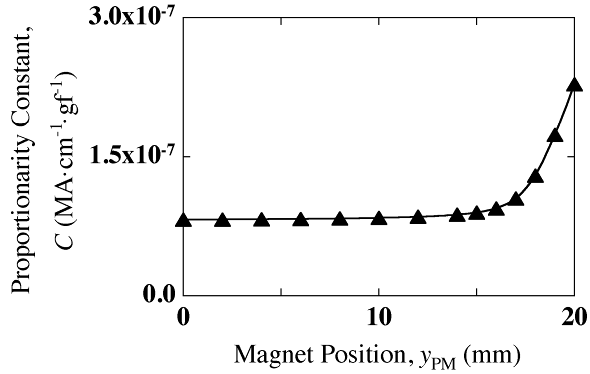


Fig. 4. Dependence of the proportionality constant C on the magnet position y_{PM}.

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